CHAPTER 13

AN ABSORBING WAVE-MAKER BASED ON DIGITAL FILTERS

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ABSTRACT

An absorbing wave maker operated by means of on-line signals from digital FIR filters is presented.

Surface elevations are measured in two positions in front of the wave maker. The reflected wave train is separated from the sum of the incident and rereflected wave trains by means of digital filtering and subsequent superposition of the measured surface elevations. The motion of the wave paddle required to absorb reflected waves is determined and added to the original wave paddle control signal.

Irregular wave tests involving test structures with different degrees of reflection show that excellent absorption characteristics have been achieved with the system.

INTRODUCTION

Coastal engineering problems are often solved by means of physical models. Physical modelling of coastal engineering phenomena requires the capability of reproducing natural conditions in the laboratory environment.
One of the problems associated with the physical modelling of water waves in laboratory wave channels is the presence of rereflected waves.
In nature the sea constitutes an open boundary which absorbs waves reflected by the coastal system.
A wave channel is a closed system: waves reflected from a model structure will be rereflected at the wave paddle, thus altering the characteristics of the wave train incident to the model structure. Consequently, the reproduction of a specified

incident wave train will often be impossible when a reflective structure is being tested.

The problem of rereflection can be reduced by applying a so-called absorbing wave maker: a combined wave generator and active wave absorber, which, in addition to generating incident waves, absorbs waves reflected from the test structure. The construction of an absorbing wave maker requires (Gilbert (1978)):

1. A means of detecting reflected waves as they approach the wave maker
2. A means of making the paddle generate waves that are, in effect, equal and opposite to the reflected waves so that the reflected waves are cancelled out as they reach the paddle. This requirement is over and above the need to generate the primary incident waves.

Milgram (1970) presented a system in which waves in a channel were absorbed by means of a moving termination at the end of the channel. The motion of the termination needed for absorption was determined by analog filtering of a surface elevation signal measured in front of the termination. This active wave absorber was not used in a combined generation and absorption mode.
Bullock and Murton (1989) described the conversion of a conventional wedge-type wave maker to an absorbing wave maker. The system developed by Bullock and Murton was based on analog filtering of a surface elevation signal measured on the face of the wave paddle. Good absorption characteristics were achieved with a less-than-perfect circuit design.

Recently, an absorbing wave maker has been installed in a wave channel at Aalborg Hydraulic Laboratory, Aalborg University.
In the following the design of this absorbing wave maker is presented and its performance is evaluated based on the results of physical model tests.

PRINCIPLE

The absorbing wave maker is operated by digital FIR filters working in real time. The relation between the input η and the output x of a digital FIR filter of length N is given by the discrete convolution integral:

$$x^k = \sum_{i=-M}^{i=M} h^i \eta^{k-i} \qquad , \quad M = \frac{N-1}{2} \tag{1}$$

where h denotes the filter impulse response (filter operator). Given a desired frequency response, the corresponding FIR filter operator is obtained by computing the inverse discrete Fourier transform of the complex frequency response function, see e.g. Karl (1989). Notice, that the filter output is delayed $\frac{N-1}{2}$ time steps relative to the input. For FIR filters operating in real time, this time delay must be removed.

The paddle displacement correction signal needed for absorption of reflected waves is determined by means of digital filtering and subsequent superposition of surface elevation signals measured in two positions in front of the wave maker (fig. 1).

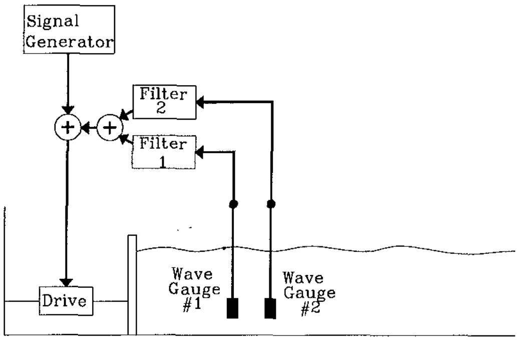

Figure 1: Principle of absorbing wave maker.

When active absorption is applied, the paddle displacement correction signal is added to the input paddle displacement signal read from the signal generator, causing the wave maker to operate in a combined generation/absorption mode. Having outlined the principle of the system, the only remaining problem in the design process is to specify the frequency response of the FIR filters applied.

FREQUENCY RESPONSE OF FIR FILTERS

In fig. 2, a wave channel equipped with two wave gauges is shown.

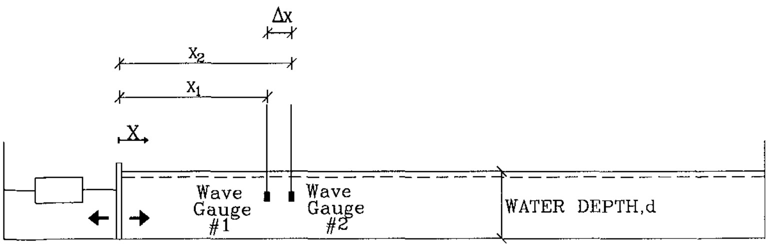

Figure 2: Wave channel with piston-type wave maker.

The surface elevation signal at a position x may be regarded as a sum of harmonic components. Considering an isolated component of frequency f, the surface elevation arising from this component may be written as the sum of the corresponding incident and reflected wave components:

$$\begin{aligned}\eta(x,t) &= \eta_I(x,t)+\eta_R(x,t)\\ &= a_I cos(2\pi ft - kx + \phi_I) + a_R cos(2\pi ft + kx + \phi_R)\end{aligned} \quad (2)$$

where

f	: frequency
$a = a(f)$	: wave amplitude
$k = k(f)$	: wave number
$\phi = \phi(f)$	: phase

and indices I and R denote incident and reflected, respectively.

Provided a linear relation exists between a given paddle displacement signal and its corresponding surface elevation signal, the paddle displacement correction signal, $X_{corr}(t)$, which cancels out the reflected component without disturbing the incident component, is given by

$$X_{corr}(t) = B \cdot a_R cos(2\pi ft + \phi_R + \phi_B + \pi) \quad (3)$$

where

B	: piston stroke/wave height relation
ϕ_B	: phaseshift between paddle displacement and surface elevation on the face of the paddle

In the following it is shown that it is possible to amplify and phase shift the surface elevation signals from the two wave gauges in such a way that their sum is identical to the paddle correction signal corresponding to absorption of the reflected component as given by eq. (3).

At the two wave gauges (fig. 2) we have:

$$\eta(x_1,t) = a_I cos(2\pi ft - kx_1 + \phi_I) + a_R cos(2\pi ft + kx_1 + \phi_R) \quad (4)$$

$$\begin{aligned}\eta(x_2,t) &= a_I cos(2\pi ft - kx_2 + \phi_I) + a_R cos(2\pi ft + kx_2 + \phi_R)\\ &= a_I cos(2\pi ft - kx_1 - k\Delta x + \phi_I) +\\ &\quad a_R cos(2\pi ft + kx_1 + k\Delta x + \phi_R)\end{aligned} \quad (5)$$

where $x_2 = x_1 + \Delta x$ has been substituted into eq. (5).

An amplification of C and a theoretical phase shift ϕ^{theo} are introduced into the expressions for $\eta(x,t)$. The modified signal is denoted η^*. For the i'th wave gauge signal the modified signal is defined as:

$$\begin{aligned}\eta^*(x_i,t) &= Ca_I cos(2\pi ft - kx_i + \phi_I + \phi_i^{theo}) +\\ &\quad Ca_R cos(2\pi ft + kx_i + \phi_R + \phi_i^{theo})\end{aligned} \quad (6)$$

This gives at wave gauges 1 and 2:

$$\begin{aligned}\eta^*(x_1,t) &= Ca_I cos(2\pi ft - kx_1 + \phi_I + \phi_1^{theo}) + \\ &\quad Ca_R cos(2\pi ft + kx_1 + \phi_R + \phi_1^{theo})\end{aligned} \tag{7}$$

$$\begin{aligned}\eta^*(x_2,t) &= Ca_I cos(2\pi ft - kx_1 - k\Delta x + \phi_I + \phi_2^{theo}) + \\ &\quad Ca_R cos(2\pi ft + kx_1 + k\Delta x + \phi_R + \phi_2^{theo})\end{aligned} \tag{8}$$

The sum of $\eta^*(x_1,t)$ and $\eta^*(x_2,t)$, which is denoted $\eta^{calc}(t)$, is:

$$\begin{aligned}\eta^{calc}(t) &= \eta^*(x_1,t) + \eta^*(x_2,t) \\ &= 2Ca_I cos(\frac{k\Delta x + \phi_1^{theo} - \phi_2^{theo}}{2}) \\ &\quad cos(2\pi ft - kx_1 + \phi_I + \frac{-k\Delta x + \phi_1^{theo} + \phi_2^{theo}}{2}) + \\ &\quad 2Ca_R cos(\frac{-k\Delta x + \phi_1^{theo} - \phi_2^{theo}}{2}) \\ &\quad cos(2\pi ft + kx_1 + \phi_R + \frac{k\Delta x + \phi_1^{theo} + \phi_2^{theo}}{2})\end{aligned} \tag{9}$$

It is seen that $\eta^{calc}(t)$ and $X_{corr}(t) = Ba_R cos(2\pi ft + \phi_R + \phi_B + \pi)$ are identical signals in case:

$$2Ccos(\frac{k\Delta x - \phi_1^{theo} + \phi_2^{theo}}{2}) = B \tag{10}$$

$$kx_1 + \frac{k\Delta x + \phi_1^{theo} + \phi_2^{theo}}{2} = \phi_B + \pi + n \cdot 2\pi,\ n \in (0, \pm 1, \pm 2, ..) \tag{11}$$

$$\frac{k\Delta x + \phi_1^{theo} - \phi_2^{theo}}{2} = \frac{\pi}{2} + m \cdot \pi,\ m \in (0, \pm 1, \pm 2, ..) \tag{12}$$

Solving eqs. (10)-(12) with respect to ϕ_1^{theo}, ϕ_2^{theo} and C with $n = m = 0$ gives

$$\phi_1^{theo} = \phi_B - k\Delta x - kx_1 + 3\pi/2 \tag{13}$$

$$\phi_2^{theo} = \phi_B - kx_1 + \pi/2 \tag{14}$$

$$C = \frac{B}{2cos(-k\Delta x + \pi/2)} \tag{15}$$

Eqs. (13)-(15) specify the frequency responses, i.e. the amplification factors and phase shifts, of FIR filters 1 and 2 in fig. 1.
Even though the theoretical frequency response of the filters easily can be calculated from the eqs. (13)-(15) one should notice that an actual realization of such

a theoretical frequence response in FIR filters might be rather difficult to obtain. The aim of this paper is not to describe design of Fir filters, but notice that the actual frequency response of the filters (read: performance of the absorbing system) are strongly dependent upon: Type of wave-maker, water depth, location of wave gauges, number of filter coefficients, sample frequency in filter etc.

PHYSICAL MODEL TEST

In order to determine the performance of the active absorption method described above, the method was implemented in the control system of a piston-type wave maker placed in a small laboratory wave channel at Aalborg Hydraulic Laboratory, Aalborg University and the method was implemented in the control system of the wedge type wave maker placed in the CIEM at LIM/UPC, Catalonia University of Technology.
The geometry of the Aalborg Hydraulics Laboratory wave channel and the wave gauge positions are given in fig. 3.

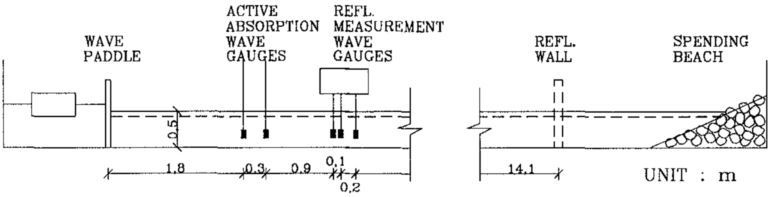

Figure 3: Wave channel and wave gauge positions.

The active absorption system was based on surface elevation measurements obtained in wave gauges positioned at distances of $x_1 = 1.80\,\text{m}$ and $x_2 = 2.10\,\text{m}$ from the wave paddle. The phase ϕ_B and gain B (see eq. (3)) were determined using the linear transfer functions derived by Biesel (1951).
When active absorption was applied, the surface elevation time series were recorded and digitized by means of a PC equipped with an A/D-D/A-card, digital filtering and superposition were performed, and the resulting paddle displacement correction signal was added to the input signal read from the signal generator.
At the far end of the channel, a spending beach was situated. In order to be able to perform tests with different degrees of reflection from the channel termination, provision was made for mounting a vertical reflecting wall in front of the spending beach.

The channel is equipped with three pairs of wave gauges mounted on a beam at distances of 3.0 m, 3.1 m and 3.3 m from the wave paddle. These gauges are used for reflection measurements.
A water depth of $d = 0.5$ m was maintained throughout the test series.

In order to evaluate the efficiency of the absorbing wave maker when applied to irregular wave tests involving test structures with different degrees of reflection, tests covering all four permutations of the alternatives

- Either with or without active absorption applied
- Either with the spending beach or the reflecting wall at the far end of the channel

were performed.
All tests were performed with exactly the same input from the signal generator: a wave paddle displacement signal corresponding to a JONSWAP-spectrum with significant wave height $H_s = 0.04$ m, peak frequency $f_p = 0.6$ Hz and peak enhancement factor $\gamma = 3.3$ sampled at a frequency of $f_s = 40$ Hz and generated by means of digital filtering of Gaussian white noise in the time domain. In each test the incident and reflected spectra were resolved as described by Mansard and Funke (1980). The incident spectra are given in fig. 4.

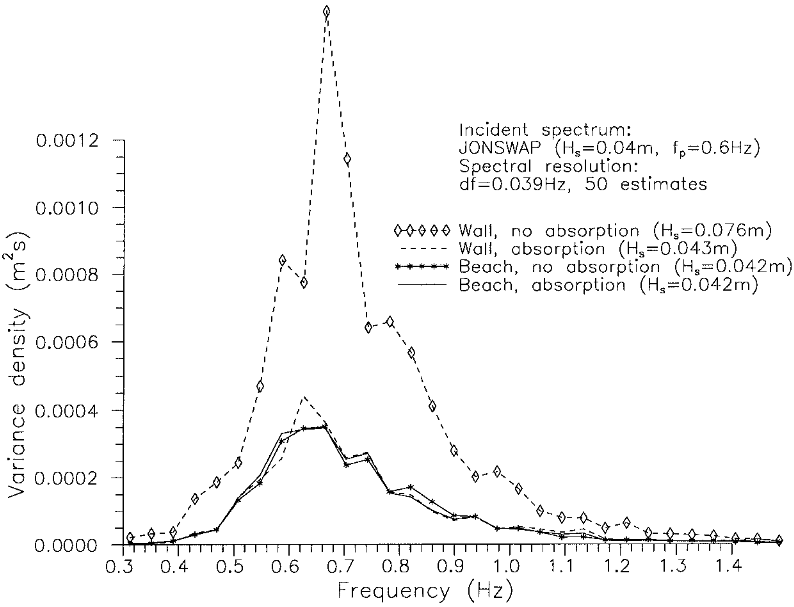

Figure 4: Incident wave spectra.

The tests showed that the spending beach reflected only 5-10% of the incoming wave energy in the frequency range of the input spectrum. Consequently, the *"beach, no absorption"* incident spectrum in fig. 4 may be regarded as the target spectrum: the disturbances introduced by rereflection are negligible.
The *"beach, absorption"* incident spectrum is almost identical to the target spectrum. This implies, that applying the active absorption system to tests involving test structures with little reflection will not introduce disturbances in the incident spectrum.
The efficiency of the active absorption system is demonstrated by the test results obtained with the reflecting wall installed at the far end of the channel. When active absorption is applied, the incident spectrum is in excellent agreement with the target spectrum, whereas the incident spectrum obtained without active absorption is significantly distorted by rereflection (see fig. 4.).

In order to visualize the effect of active absorption in the time domain, the following test was performed: the reflecting wall was installed in the far end of the channel, and irregular waves were generated. After 60 seconds, wave generation was terminated, and active absorption was applied. A surface elevation time series recorded at $x = 3.0\,\mathrm{m}$ is given in fig. 5 a.
For comparison, a time series recorded in a similar test in which active absorption was not applied is given in fig. 5 b.

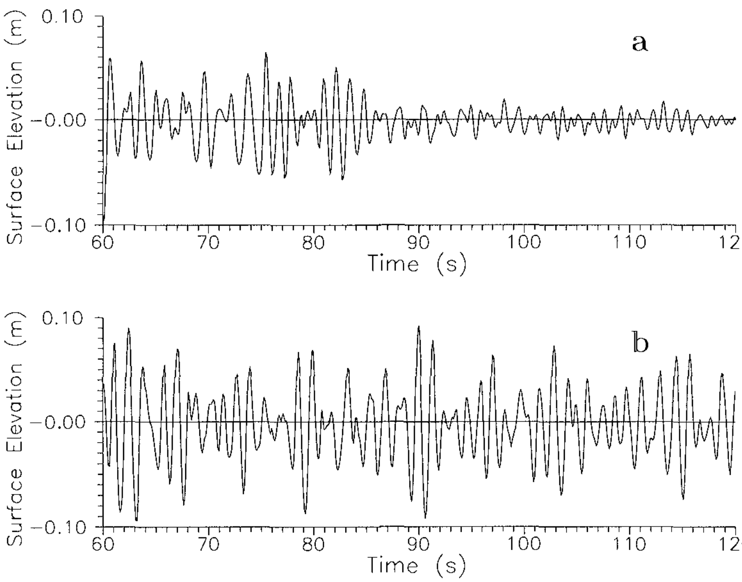

Figure 5: Time series obtained with (a) and without absorption (b).

Furthermore, the stability of the system was tested.
Again, the reflecting wall was mounted at the far end of the channel. The active absorption system was applied, and a paddle displacement time series of length $T = 51.2\,\mathrm{s}$ was generated, and sent repeatedly to the wave maker.
Two surface elevation time series of length T were recorded starting from $t = T$ and $t = 25T$, respectively, and the incident and reflected spectra were resolved. The resulting incident spectra are given in fig. 6.

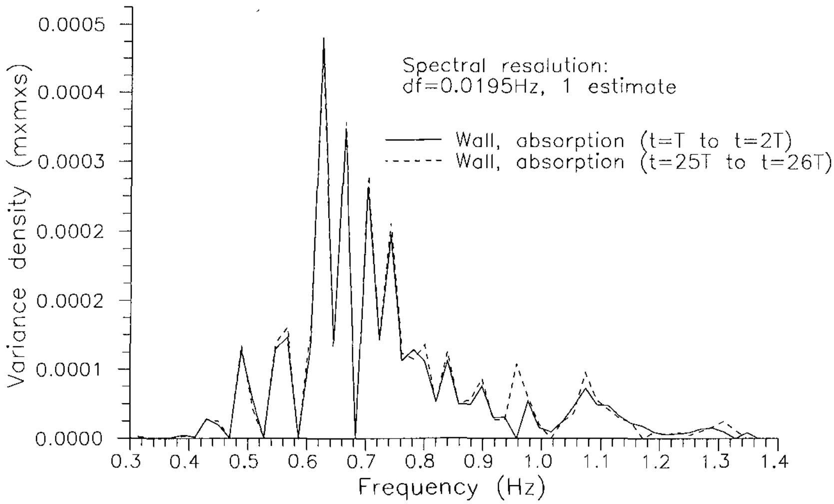

Figure 6: Incident wave spectra.

Fig. 6 indicates that the system is stable. Apparently, 25 repetitions of the input signal (approx. 20 minutes of wave generation) have not caused significant disturbance in the incident spectrum despite reflection from the wall at the channel termination.

The geometry of the wave channel in the CIEM at LIM/UPC and the wave gauge positions are given in fig. 7.

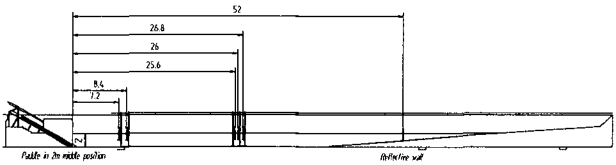

Figure 7: Wave channel and wave gauge positions.

In this test, the reflection compensation system was based on surface elevation measurements obtained in wave gauges positioned at distances of $x_1 = 7.20m$ and $x_2 = 8.40m$ from the wave paddle. The phase ϕ_B and gain B were determined using the linear transfer functions derived by Biesel (1951). An additional phase shift was introduced in order to compensate for a measured time delay of $0.1s$ between demand and feedback signals in the wave maker control system.
When reflection compensation was applied, the surface elevation time series were recorded and digitised by means of a PC equipped with an A/D-D/A-card, digital filtering and superposition was performed, and the resulting paddle displacement correction signal was added to the input signal read from the signal generator.
At the far end of the channel, a spending beach was situated. In order to be able to perform tests with different degrees of reflection from the channel termination, a vertical reflecting wall could be placed in front of the spending beach.
The channel was equipped three wave gauges mounted at distances of 25.6m, 26.0m and 26.8m from the wave paddle. These gauges were used for reflection measurements.
A water depth of $d = 2.0m$ was maintained throughout the test series.
All tests were performed with exactly the same input from the signal generator: a wave paddle displacement signal corresponding to a JONSWAP-spectrum with significant wave height $H_s = 0.25m$, peak period of $T_p = 3s$ and peak enhancement factor $\gamma = 3.3$ sampled at a frequency of $f_s = 20Hz$ and generated by means of digital filtering of Gaussian white noise in the time domain.
In each test, the incident and reflected spectra were resolved as described by Mansard and Funke (1980).

The spending beach had a reflection coefficient of only 6-8 % in the frequency range of the input spectrum. Consequently, the incident spectrum measured with the spending beach at the far end of the channel and no reflection compensation applied may be regarded as the target spectrum: the disturbances introduced by rereflection are negligible.

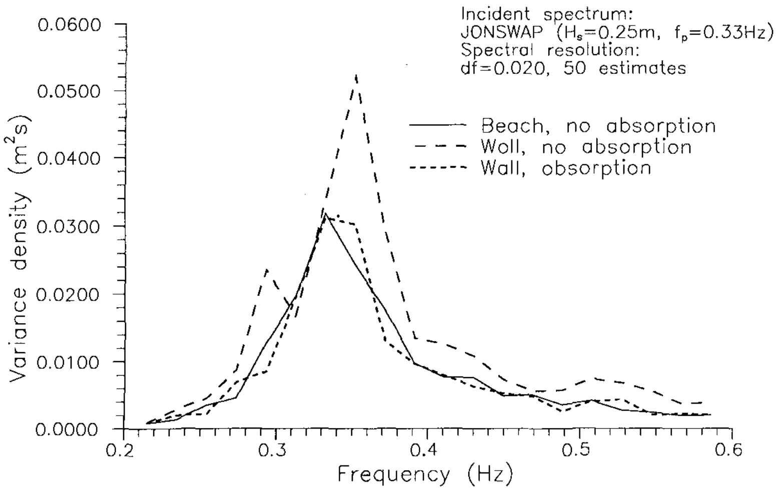

Figure 8: Incident wave spectra.

In fig. 8 the efficiency of the active absorption system is demonstrated. The figure shows that the tests in the large flume are very comparable to the tests in the small flume.

The performance of the reflection compensation system developed at Aalborg University (AU) appears to be excellent. When the system is applied to the test with a spending beach at the channel termination, the measured incident spectrum is almost identical to the target spectrum (fig. 4 and fig. 8). This implies, that applying this reflection compensation system to tests involving test structures with little reflection will not introduce disturbances in the incident spectrum.

LONG WAVES

Laboratory tests with irregular waves often give problems with absorption of long waves. Absorption of long waves requires an enormous stroke of the paddle. This means, that the designer of the filters always have to limit the low frequent performance of the system in order to have enough stroke in the wave maker system. Fig. 9 shows the performance of the system used in the tests at Aalborg Hydraulics Laboratory. Even though the figure indicates a rather poor performance of the system for long waves, it should be noted that as long as the system absorps a part of the re-reflected waves it will prevent the growth of long waves.

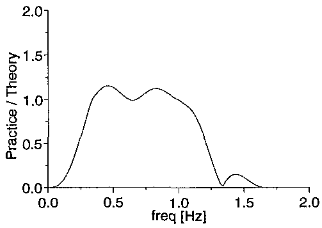

Figure 9: Performance of system.

CONCLUSION

A method for active absorption of reflected waves in wave channels by means of an absorbing wave maker has been presented. (Notice, that the system also can be very usefull in numerical models).
The motion of the wave maker, which is needed for absorption, is determined by means of digital filtering and subsequent superposition of surface elevations measured in two fixed positions in front of the wave maker.
The method has been implemented in the control system of a piston-type wave maker installed in a wave channel, and irregular wave tests involving test structures with different degrees of reflection have been carried out in order to determine the performance of the absorbing wave maker.
The tests performed imply that excellent absorption characteristics have been achieved. The absorbing wave maker is capable of reducing the problem of rereflection considerably even at very high levels of reflection. Furthermore, the active absorption system appears to be stable.
Converting a conventional wave maker to an absorbing wave maker based on the method presented above is relatively inexpensive considering the improvements achieved: the only requirements are two conventional wave gauges and a PC

equipped with an A/D-D/A-card. These facilities will normally already be available in most laboratory environments (if a PC equipped with an A/D-D/A-card is used as signal generator for the wave maker, the wave gauges can be connected to this computer, allowing the computer to perform signal generation and correction signal calculation simultaneously).

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